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CHARACTERISTICS OF A LONGITUDINAL GLOW DISCHARGE

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A system of equations has been solved that describes the positive column in a glow discharge in a cylindrical channel bearing a longitudinal gas flow.

Glow discharges (GD) have been widely used recently in electronics, various technological processes, plasmochemical reactors, and so on. It is therefore important to examine the distributions of the internal GD parameters such as electron concentration, electric field strength, and neutral-particle temperature as affected by external conditions. Some regularities have been established in the electric fields and electron concentrations in longitudinal GD [1-4]. Measurements have been made [5-8] on the temperature patterns in axially symmetrical discharges bearing longitudinal gas flows. In those papers, the neutral-particle temperature in the positive GD column was calculated by solving the energy-conservation equation with a given distribution for the internal heat sources over the positive column. However, in a GD in a gas flow, the output from the internal heat sources varies along the axis and is in fact an unknown function to be determined. It is much more complicated to determine the parameters E, n_e, and T together. In [9], a solution was obtained numerically, and the distributions of the parameters in flowing hydrogen were obtained for certain conditions. However, it is preferable to derive analytic solutions in order to elucidate the general regularities in convective heat transfer in a glow discharge, and these are also useful in checking and improving numerical-calculation programs for more complicated cases.

In the proposed model, the positive column in a cylindrical channel is considered in relation to three forms of particle: neutral particles, electrons, and singly charged positive ions. The following form can be given [10, 11] to the stationary equations of continuity for the electrons and positive ions:

$$\operatorname{div}\vec{\Gamma}_i = \nu n_e - \delta n_e n_i, \tag{1}$$

$$\operatorname{div}\vec{\Gamma}_e = \nu n_e - \delta n_e n_i. \tag{2}$$

The following equations describe the charged-particle flux densities across any area in the discharge zone:

$$\vec{\Gamma}_i = n_i\vec{v} - D_i\nabla n_i + n_i\mu_i\vec{E}, \tag{3}$$

$$\vec{\Gamma}_e = n_e\vec{v} - D_e\nabla n_e - n_e\mu_e\vec{E}, \tag{4}$$

where the first term on the right incorporates the convective charge transport, the second arises from diffusion, and the third from electric-field drift. We add and subtract (1) and (2) term by term and use the condition for plasma quasineutrality ($n_i \approx n_e = n$) with (3) and (4) to get

$$2\operatorname{div}(n\vec{v}) - \operatorname{div}[(D_e + D_i)\nabla n] + (\mu_i - \mu_e)n\operatorname{div}\vec{E} + (\mu_i - \mu_e)\cdot\vec{E}\nabla n = 2\nu n - 2\delta n^2, \tag{5}$$

$$\operatorname{div}[(D_e - D_i)\nabla n] + (\mu_i + \mu_e)\vec{E}\nabla n + (\mu_i + \mu_e)n\operatorname{div}\vec{E} = 0. \tag{6}$$

It follows from (6) and (5) that

$$\operatorname{div}(n\vec{v}) = \operatorname{div}(D_a\nabla n) + \nu n - \delta n^2, \tag{7}$$

where $D_a = (D_e\mu_i + D_i\mu_e)/(\mu_e + \mu_i)$.

If the condition $D_a/R^2 \gg n\delta$ is obeyed at any point in the discharge zone, bulk recombination is less important than ambipolar charge diffusion to the wall, and it can be neglected.

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Some estimates [12] indicate that diffusion-limited conditions apply for a positive column in nitrogen in the reduced-pressure range pR = 3-40 torr·cm and currents I/R = 20-160 mA/cm. Similar conditions occur for hydrogen [9] for Ip < 4 A·torr. We assume also that the gas speed has only an axial component and is constant (D_a = constant), while charge transport by diffusion along the axis is negligible. These assumptions have been used previously in [2-6, 13], and then (7) is written in a cylindrical coordinate system as

$$\mathrm{Pe}_d\,\frac{\partial\bar{n}}{\partial z}=\frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial\bar{n}}{\partial r}\right)+\beta\bar{n}. \tag{8}$$

Here

$$r=\frac{r_1}{R}\ ;\ z=\frac{z_1}{R}\ ;\ \mathrm{Pe}_d=\frac{vR}{D_a};\ \beta(z)=\frac{\nu(z)R^2}{D_a};\ \bar{n}(r,\ z)=\frac{n(r,\ z)}{n(0,\ 0)}.$$

The current, field strength, and electron concentration in the positive column are related by Ohm's law in integral form:

$$I=2\pi e\mu_e E n(0,\ 0)R^2\int\limits_0^1\bar{n}r\,dr. \tag{9}$$

The ionization frequency is a function of E/N, so (8) and (9) are coupled. The following power-law relationship [14] is an adequate approximation for the dependence of the ionization frequency on E/N within fairly wide limits:

$$\frac{\nu}{N}=\alpha\left(\frac{E}{N}\right)^m, \tag{10}$$

where α and m are dependent on the nature of the gas.

At moderate gas speeds (M < 0.25), one can neglect the viscous energy dissipation, the energy transport by radiation, the change in gas kinetic energy, and the molecular transport of heat along the axis by comparison with the convective energy transport, the dual energy dissipation from the electric field, and the heat transport in the radial direction by thermal conduction [15]. Using these assumptions and assuming that the physical properties of the gas are constant, we neglect V–T relaxation and write the energy-conservation equation for the neutral particles in the form

$$\mathrm{Pe}\,\frac{\partial\theta}{\partial z}=\frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\frac{\partial\theta}{\partial r}\right)+\frac{R^2E^2e\mu_e n\eta}{\varkappa}, \tag{11}$$

where Pe = vR/a; $\theta = T - T_R$, and η is the fraction of the discharge energy going directly to increase the translation of gas temperature. Then $\eta \approx 1$ for atomic gases. In a molecular gas, much of the discharge energy goes to excite the molecular vibrational levels. The heating due to vibrational relaxation can be neglected if the V–T relaxation time greatly exceeds the time spent by the molecules in the discharge zone.

The assumptions of constant physical properties and constant gas speed restrict the application of the results to discharges in which the temperature changes in the neutral gas are small. In [7], the calculated temperature patterns were compared for the cases where the physical properties of the gas were constant and where they were dependent on temperature. The calculations showed that the results were virtually the same if $q_V \ll 1$ W/cm³.

The system (8)-(11) describes the positive column in a cylindrical channel containing a flowing gas subject to the above assumptions, and these equations should be supplemented with the conditions of uniqueness

$$\begin{gathered}\bar{n}(r,\ 0)=\varphi(r),\ \bar{n}(1,\ z)=0,\ \bar{n}_r(0,\ z)=0,\\ \Theta(r,\ 0)=\Theta_0(r),\ \Theta(1,\ z)=0,\ \Theta_r(0,\ z)=0,\end{gathered} \tag{12}$$

where $0\leqslant r\leqslant 1,\ z\geqslant 0$. Then the derivation of the internal parameters amounts to solving (8) and (9), which give E and n. Then these E and n are used to solve (11), which defines the neutral-particle temperature pattern. We solved (8) and (9) by the method used in [4] to get formulas for the field strength and electron concentration:

$$E(z) = E_\infty \left(\frac{\lambda_1^2 m Q^m}{\mathrm{Pe}_d} \int\limits_0^z Q^{-m} dz + \frac{E_\infty^m Q^m}{E_0^m Q_0^m} \right)^{-\frac{1}{m}}, \tag{13}$$

$$n(r,\ z) = \frac{I \sum\limits_{n=1}^{\infty} A_n J_0(\lambda_n r) \exp(-\lambda_n^2 z/\mathrm{Pe}_d)}{\pi e \mu_e R^2 E(z) Q(z)}. \tag{14}$$

Here

$$E_\infty = \left(\frac{\lambda_1^2 D_a N^{m-1}}{R^2 \alpha} \right)^{\frac{1}{m}};\ Q(z) = 2 \sum_{n=1}^{\infty} \lambda_n^{-1} J_1(\lambda_n) \exp(-\lambda_n^2 z/\mathrm{Pe}_d);$$

$$A_n = 2J_1^{-2}(\lambda_n) \int\limits_0^1 r J_0(\lambda_n r) \varphi(r)\, dr.$$

To solve (11), we use a finite Khankel integral transformation with respect to the variable r:

$$\tilde{\Theta}(\lambda_i,\ z) = \int\limits_0^1 r J_0(\lambda_i r) \Theta(r,\ z)\, dr. \tag{15}$$

We convert from the transform $\tilde{\Theta}$ to the original Θ via

$$\Theta(r,\ z) = \sum_{i=1}^{\infty} \frac{2J_0(\lambda_i r)}{J_1^2(\lambda_i)} \tilde{\Theta}(\lambda_i,\ z). \tag{16}$$

We apply (15) to (11) and use (12) together with (13) and (14) to get

$$\mathrm{Pe} \frac{d\tilde{\Theta}}{dz} = -\lambda_i^2 \tilde{\Theta} + \frac{IE(z)\eta}{\varkappa\pi Q(z)} \int\limits_0^1 r J_0(\lambda_i r) \left[\sum_{n=1}^{\infty} A_n J_0(\lambda_n r) \exp(-\lambda_n^2 r/\mathrm{Pe}_d) \right] dr. \tag{17}$$

The second term on the right in (17) is different from zero for i = n, which follows from the orthogonality of Bessel functions. Then the solution to (17) is put as

$$\tilde{\Theta}(\lambda_i,\ z) = \exp(-\lambda_i^2 z/\mathrm{Pe}) \left[\int\limits_0^1 \Theta_0(r) J_0(\lambda_i r)\, r dr + \frac{\lambda_n J_1(\lambda_n) I\eta}{\varkappa\pi\, \mathrm{Pe}} \int\limits_0^z E(z) \exp(\lambda_n^2 z/\mathrm{Pe})\, dz \right]. \tag{18}$$

Substituting (18) into (16) gives the solution to (11):

$$\Theta(r,\ z) = \sum_{i=1}^{\infty} B_i J_0(\lambda_i r) X_i + \frac{I\eta}{2\varkappa\pi} \sum_{n=1}^{\infty} \frac{\lambda_n J_0(\lambda_n r)}{J_1(\lambda_n)} X_n K_n, \tag{19}$$

where

$$B_i = 2J_1^{-2}(\lambda_i) \int\limits_0^1 \Theta_0 J_0(\lambda_i r)\, r dr;\ X_n = \exp(-\lambda_n^2 z/\mathrm{Pe});$$
$$K_n = \frac{1}{\mathrm{Pe}} \int\limits_0^z E(z) X_n^{-1} dz. \tag{20}$$

Then formulas (13), (14), and (19) give correspondingly the field strength, electron concentration, and neutral-particle temperature. In the particular case $\varphi(r) = I_0(\lambda_1 r)$, $\Theta(r,\ 0) = 0$ the working formulas for these parameters become

$$E(z) = E_\infty [1 + (\Omega^m - 1) \exp(-m\lambda_1^2 z/\mathrm{Pe}_d)]^{-\frac{1}{m}}, \tag{21}$$

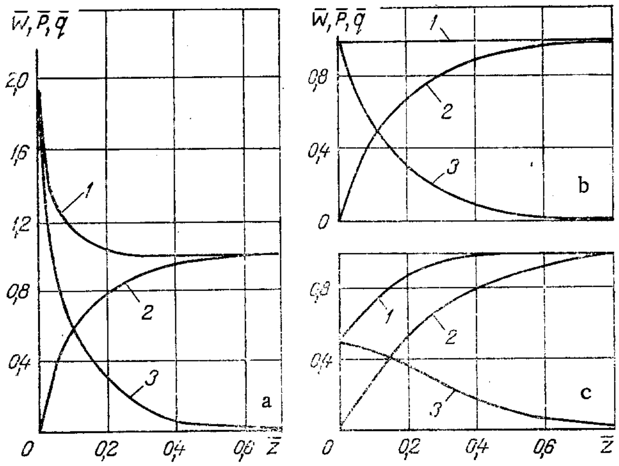

Fig. 1. Distributions of the fractions of heat production $\bar{p}$ (1), heat transfer to the wall $\bar{q}$ (2), and energy $\bar{W}$ (3) consumed in heating gas along the length of the discharge channel: a) Ω = 0.5; b) 1; c) 2; m = 2; Le = 1.

$$n(r,\ z) = J_0(\lambda_1 r)\frac{E_0}{E(z)}, \tag{22}$$

$$\Theta(r,\ z) = \frac{IJ_0(\lambda_1 r)\lambda_1\eta}{2\pi\varkappa J_1(\lambda_1)\,\mathrm{Pe}}\exp(-\lambda_1^2\bar{z})\int_0^z E(z)\exp(\lambda_1^2\bar{z})\,dz, \tag{23}$$

where $\Omega = E_\infty/E_0$. It has been shown [4] that the field strength can vary in three ways along the length, in accordance with the value of Ω. If Ω = 1, then $E = E_\infty = \text{const}$. In the other cases, E increases or decreases as z increases and tends to the limiting value E_∞ correspondingly for Ω > 1 and Ω < 1. The output from the internal heat sources per unit length P = IE(z) varies similarly. We substitute (21) into (23) to get

$$\Theta(r,\ z) = \frac{\eta IE_\infty J_0(\lambda_1 r)X^{-1}}{2\pi\varkappa\lambda_1 J_1(\lambda_1)}\int_1^X X^{\mathrm{Le}}(X^{m\mathrm{Le}} + \Omega^m - 1)^{-\frac{1}{m}}dX, \tag{24}$$

where $X = \exp(\lambda_1^2\bar{z})$; $\mathrm{Le} = \mathrm{Pe}/\mathrm{Pe}_d = D_a/a$.

It follows from (24) that the temperature distribution in the positive column is dependent on parameter m, the Lewis–Semenov number Le and the number Ω. The expression in the integral in (24) is a differential binomial. The integral from it can be expressed in terms of elementary functions only in certain cases. With m = 2 and Le = 1, (24) becomes

$$\Theta(r,\ z) = \frac{\eta IE_\infty J_0(\lambda_1 r)}{2\pi\varkappa\lambda_1 J_1(\lambda_1)}\{[1 + (\Omega^2 - 1)X^{-2}]^{0.5} - \Omega X^{-1}\}. \tag{25}$$

From (25) we can determine the relative heat flux to the wall per unit chamber length:

$$\bar{q} = \frac{q}{IE_\infty\eta} = [1 + (\Omega^2 - 1)X^{-2}]^{0.5} - \Omega X^{-1}. \tag{26}$$

The integral energy-balance equation for the neutral gas derived from (11) is as follows in dimensionless form:

$$\bar{W} = \bar{P} - \bar{q}, \tag{27}$$

where

$$\bar{W} = \frac{\pi\varkappa\,\mathrm{Pe}}{IE_\infty\eta}\,\frac{d\Theta_c}{dz};\ \bar{P} = E/E_\infty.$$

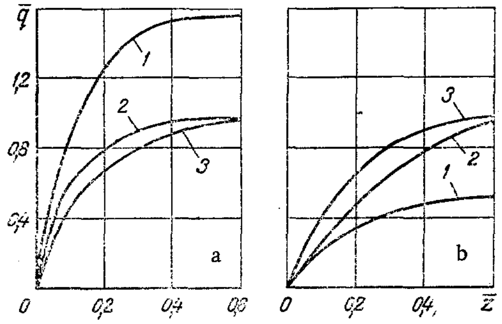

Fig. 2. Distribution of the relative heat transfer to the wall $\bar{q}$ along the chamber length: a) Ω = 0.5; b) 2; 1) Le = 0.05; 2) 1; 3) 10.

The left-hand side of this equation is the fraction of the energy used in heating the gas. The first term on the right is the fraction of the energy deposited in the positive column, while the second is the fraction given up to the wall. Formulas (21), (26) and (27) enable one to calculate the energy balance for various values of Ω. The calculations and the graphs in Fig. 1 show that the length of the limiting part in which the gas is heated increases with Ω; the relative heat flux to the wall at the same $\bar{z}$ decreases, while the fraction of the energy consumed in heating the gas decreases more smoothly as $\bar{z}$ increases.

In this case, the Lewis–Semenov number characterizes the rates of change in the electron concentration and field strength along the discharge chamber relative to the temperature pattern of the neutral particles [16]. It follows from (24) that the neutral-particle temperature pattern is independent of Le and m for $\Omega = 1$. If $\Omega \neq 1$ and Le $\to 0$, n and E vary along the chamber much more slowly than do the thermal characteristics. For this case we get from (24) that

$$\Theta(r,\ z) = \frac{\eta I E_0 J_0(\lambda_1 r)}{2\lambda_1 \pi \varkappa J_1(\lambda_1)} [1 - \exp(-\lambda_1^2 \bar{z})]. \tag{28}$$

This shows that the temperature pattern will be determined by the power density from the internal heat source IE_0 in the initial section.

When Le $\to \infty$, the electrical parameters vary along the length much more rapidly than the thermal ones. It follows from (24) for this limiting case that

$$\Theta(r,\ z) = \frac{\eta I E_\infty J_0(\lambda_1 r)}{2\lambda_1 \pi \varkappa J_1(\lambda_1)} [1 - \exp(-\lambda_1^2 \bar{z})]. \tag{29}$$

The temperature pattern is here determined by the power density of the internal heat source in the limiting part IE_∞.

In the general case, the way Le influences the thermal characteristics is dependent on Ω. Numerical calculations have been performed on the relative heat flux to the wall for various values of Le via (21) and are shown in Fig. 2, which indicates that the relative heat flux to the wall increases with Le if $\Omega > 1$ but decreases if $\Omega < 1$. The neutral-particle temperature varies analogously.

NOTATION

r_1 and z_1, radial and longitudinal coordinates; R, chamber radius; I, current; $\vec{E}$, electric field strength; $\vec{v}$, gas velocity; p, gas pressure; N and T, density and temperature of neutral particles; T_c, neutral particle temperature averaged over the cross section of the discharge chamber; T_R, temperature of the discharge chamber wall; q, heat flux per unit length of the discharge chamber; q_V, density of the internal heat source; a and $\varkappa$, thermal diffusivity and thermal conductivity of gas; n_e, μ_e, and e, electron concentration, mobility, and charge; n_i and μ_i, concentration and mobility of ions; ν, ionization frequency; δ, recombination coefficient; D_a, ambipolar diffusion coefficient; Pe, Peclet number; Pe_d, Peclet diffusion number; M, Mach number; $\bar{z} = z/Pe$; DC, discharge chamber; E_0 and E_∞, electric fields in the section z = 0 and in the limiting section.

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